Evaluation of Effectiveness of Education of Mathematics in High Schools on the Example of Schools in the City of Wroclaw

Marek Biernacki

Department of Mathematics, Wroclaw University of Economics, Poland
Email.: marek.biernacki@ue.wroc.pl

Doi:10.5901/mjss.2013.v4n11p523

Abstract

This work is a continuation of the article by Biernacki, [Biernacki (2012)] which evaluated the educational system in Poland in four areas:
1. Economic, in which service performance of a school is assessed (expressed by comparing the production contribution and the economic effect, i.e. graduates).
2. Professional, which assesses the effectiveness of teaching, which is measured by the ability to improve knowledge and skills of the student.
3. Personal, related to student's, pupil's and parent's satisfaction understood as the quality (subjective) of teaching.
4. Social, which assesses increase of social welfare resulting from the activities of the school.

In this work a synthetic measure is proposed to evaluate a school in the distinguished aspects, but also taking into consideration the fact that assessment of schools was based on three of them (based on available data).

Keywords: effectiveness, education system.

1. Introduction

Effective functioning of public institutions, especially in the area of education is included in the basic problem of justice of the entire the socio-economic system, in which the starting point is the human being - their value and dignity. Another important factor maintaining effective education is human capital, which according to many concepts is a factor of economic growth of the whole country [Mankiw, N., Romer, D., Weil, D. (1992)]. In this time of economic crisis and high public debt in Poland there is no real chance to increase spending on education. Consequently, what should be improved is managing of education system's entities. Better management forces greater effectiveness of the organization. To achieve the above one needs to evaluate and control activities.

The aim of this article is to present a multidimensional evaluation of public institutions in the education sector for the purpose of optimizing costs and improving quality of service.

2. Economic aspect

Performance level of the education system at the secondary level in Poland, measured by the ratio of the average result of PISA exam to expenditure per pupil is among the highest in the analyzed countries [see Biernacki (2012)].

According to Barro and Lee [Barro, Lee (2001)] educational performance in science subjects is the most significant for economic growth. Therefore, the area of analysis will be the results of education of mathematics in secondary schools. Effectiveness of mathematics education at the secondary level in Poland is in the years 2003 - 2009 as compared to the international PISA exam is at a constant rate equal to the average result of assessed students from OECD countries. Whereas the proportion of the weakest Polish students (in OECD – PISA research who do not exceed 1st and 2nd level of difficulty of tasks) is fixed at 20%, and the percentage of best students, those who solve tasks at 5th and 6th levels of difficulty is at a constant low level of 10%.

Rating of a school's economic area was carried out using a non-parametric method - DEA (Data Envelopment Analysis) [Charnes, Cooper and Rhodes (CCR), 1978]. From a statistical approach, based on central tendency observed in the sample, this rating is different in that it assesses the effectiveness of each object not on the grounds of average objects but taking into consideration the best ones – the most effective ones. The set of points in a sample is surrounded by area connecting the best objects, hence the name of the method: data envelopment analysis. DEA method identifies
efficient objects, the ones which produce in the most economical way under given circumstances. Thus, it is a measure of effectiveness of the relative dependable on the analyzed data set. The analysis can be focused on maximizing effects or minimizing consumption of resources. A measure of (in)efficiency of an object reflects its distance from the boundary area. The effectiveness of the school is obtained by maximizing the ratio of weighted services to weighted inputs, provided that the same indicators for each school are less than or equal 1. That is:

$$\max_{j_0} = \frac{\sum_{r=1}^{s} u_{rj_0} y_{ij_0}}{\sum_{j=1}^{m} v_{ij_0} x_{ij_0}}$$

provided that

$$\frac{\sum_{r=1}^{s} u_{rj_0} y_{ij_0}}{\sum_{j=1}^{m} v_{ij_0} x_{ij_0}} \leq 1; j = 1, \ldots, n$$

where: $y_{ij}$ - r-th service produced by j-th school, $x_{ij}$ - investment i used by the school j, $u_{rj}$, $v_{ij} \geq 0$ – weights which are determined by solving the above formula

3. Professional aspect (perspective)

From a praxeological point of view (purpose of the action) to the state and society should aim at not only maintaining the average level of education, or to adapt to the level of the OECD and the EU, but also appreciate gifted students who in the future will build an economy that is based on knowledge, and minimize the proportion of the weakest students. Sparkers J., [1999] showed that there is a strong correlation between poor academic performance and poverty. Increase in the level of knowledge of poor children gives them a chance to escape poverty. On the one hand, the lower the population’s poverty, the lower the inequality of income distribution. On the other hand, the lower the number of the poor, the lower the value of social benefits, and a smaller number of thefts, robberies, i.e. greater social well-being.

Assessment of professional dimension was carried out with the use of the method of Educational Value Added (EVA), which like the DEA is a relative evaluation method. The simplest example in a class of linear statistical models is the model of “education production function” expressed by the formula (Goldstein and Spiegelhalter):

$$Y_{ij} = \beta x_{ij} + \gamma z_{j} + u_{j} + e_{ij}$$

where: $Y_{ij}$ is a quantitative result of i-th student in j-th university (on j-th faculty),
$x_{ij}$ - corresponding characteristics of the student,
$z_{j}$ - resources (measures) of the j-th university (faculty),
$u_{j}$ - measures the relative efficiency of j-th university (faculty),
e_{ij} - is the random error assigned to i-th student in j-th university,
$\beta$ and $\gamma$ are vectors of constant coefficients associated with $x_{ij}$ and $z_{j}$ respectively.

By estimating the vectors, and we can estimate the effectiveness of j-th school.

Figure 1. shows average EVA results in mathematics for secondary schools in Wroclaw during 2010-2012. From the data one can observe the impact of competition on the outcome of education. Most of the ‘weakest’ schools in 2010 have improved over time their education score (EVA) in mathematics “escaping” from the end of the rankings. Observation of the outcomes of education may allow the management and staff of the school to find optimal conditions for such an allocation of resources, that will provide the best educational results, and help students and their parents
choose the right school.
In order for the score of this aspect to be in the interval \([0, 1]\) a standardization is made:

\[
u_j = \frac{u_j - \min_j u_j}{\max_j u_j - \min_j u_j}
\]

**Figure 1.** EVA results in mathematics for secondary schools in Wroclaw in the years 2010 to 2012.

**Source:** Own calculations using Excel based on data from the Regional Examination Commission in Wroclaw (2010, 2011, 2012).

4. **Social aspect (perspective)**

In order to assess the level of social welfare the so-called abbreviated Sen's welfare functions were used, they are expressed as: \(V = \mu (1 - G)\), where \(\mu\) is the average income, and \(G\) is the Gini's coefficient. At the outset and completion of the education process there are presented vectors of students' characteristics. When evaluating secondary schools they are the number of points gained at the lower secondary school final exam and points gained at the matriculation examination – respectively, \(\bar{x}^0 = [x_1^0, x_2^0, \ldots, x_n^0]\), \(\bar{x}^1 = [x_1^1, x_2^1, \ldots, x_n^1]\). Then, using a abbreviated Sen's welfare function one calculates the change in social welfare \(SWF\) which is the result of the activity of the assessed school. After modifications, an index was obtained as:

\[
SWF = \frac{\bar{x}^{-1} (1 - G_1) - \bar{x}^0 (1 - G_0) + 1}{2}
\]

where \(G_0, G_1\) are the values of the Gini's indices of these distributions.

5. **Synthetic measure**

Each school can be assigned a point of the cube \(I^3\) (vector of three coordinates in the range \([0, 1]\). Simplest synthetic measure that can evaluate a school is weighed average. In the proposed assessment of schools in Wroclaw an arithmetic mean was included into the synthetic evaluation.

Table 1. presents the results of synthetic index of Wroclaw secondary schools assessment, which is the arithmetic mean of the indices: economic area (DEA), the area of the praxeological internal (EVA) and the external praxeological area, i.e. social dimension (Social Measure.)

<table>
<thead>
<tr>
<th>School</th>
<th>Number of graduates</th>
<th>Average G-MP</th>
<th>Cost of employment</th>
<th>Students / Teachers</th>
<th>Average M-M</th>
<th>EVA</th>
<th>DEA</th>
<th>Social Measure</th>
<th>Synthetic average</th>
<th>Ran-ki</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOI</td>
<td>177</td>
<td>68.12</td>
<td>4 854.96</td>
<td>44.36</td>
<td>59.25</td>
<td>0.47</td>
<td>1.00</td>
<td>0.564</td>
<td>0.677</td>
<td>5</td>
</tr>
<tr>
<td>LOII</td>
<td>229</td>
<td>77.27</td>
<td>5 240.81</td>
<td>44.29</td>
<td>63.29</td>
<td>0.20</td>
<td>0.93</td>
<td>0.556</td>
<td>0.561</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOIII</td>
<td>171</td>
<td>95,70</td>
<td>5162,44</td>
<td>39,13</td>
<td>91,61</td>
<td>0,88</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOIV</td>
<td>188</td>
<td>75,88</td>
<td>5339,09</td>
<td>47,36</td>
<td>64,69</td>
<td>0,36</td>
<td>0,97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOV</td>
<td>136</td>
<td>85,15</td>
<td>5204,58</td>
<td>36,17</td>
<td>72,88</td>
<td>0,51</td>
<td>0,93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOVI</td>
<td>140</td>
<td>66,27</td>
<td>5089,84</td>
<td>39,55</td>
<td>54,67</td>
<td>0,35</td>
<td>0,97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOVII</td>
<td>273</td>
<td>90,42</td>
<td>5328,51</td>
<td>53,85</td>
<td>79,50</td>
<td>0,30</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOVIII</td>
<td>171</td>
<td>82,36</td>
<td>5188,88</td>
<td>56,44</td>
<td>71,37</td>
<td>0,45</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOIX</td>
<td>241</td>
<td>83,80</td>
<td>5235,15</td>
<td>57,93</td>
<td>74,38</td>
<td>0,41</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOX</td>
<td>234</td>
<td>74,80</td>
<td>5218,51</td>
<td>53,42</td>
<td>62,28</td>
<td>0,27</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXI</td>
<td>87</td>
<td>61,75</td>
<td>5017,06</td>
<td>31,29</td>
<td>48,09</td>
<td>0,18</td>
<td>0,98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXII</td>
<td>244</td>
<td>87,62</td>
<td>5476,59</td>
<td>33,38</td>
<td>77,52</td>
<td>0,49</td>
<td>0,94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXIII</td>
<td>254</td>
<td>80,53</td>
<td>4988,76</td>
<td>48,11</td>
<td>73,42</td>
<td>0,65</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXIV</td>
<td>147</td>
<td>93,63</td>
<td>4872,80</td>
<td>34,35</td>
<td>91,90</td>
<td>1,00</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXV</td>
<td>198</td>
<td>70,46</td>
<td>5108,33</td>
<td>48,77</td>
<td>58,92</td>
<td>0,32</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXVI</td>
<td>53</td>
<td>55,89</td>
<td>4985,97</td>
<td>34,64</td>
<td>37,77</td>
<td>0,00</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXVII</td>
<td>176</td>
<td>72,25</td>
<td>4926,75</td>
<td>47,31</td>
<td>62,52</td>
<td>0,44</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXXIV</td>
<td>66</td>
<td>59,64</td>
<td>5240,90</td>
<td>31,73</td>
<td>43,00</td>
<td>0,05</td>
<td>0,95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXXIX</td>
<td>32</td>
<td>54,00</td>
<td>4940,39</td>
<td>25,81</td>
<td>42,06</td>
<td>0,29</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOXXX</td>
<td>43</td>
<td>47,63</td>
<td>5254,33</td>
<td>14,53</td>
<td>34,47</td>
<td>0,21</td>
<td>1,00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Source:** Own calculations based on data from Wroclaw Educational Authority, 2011.

Wherein: Average G-MP is the average number of lower secondary school points from mathematics and natural sciences, and the average M-M is the average number of points obtained in the high school final exam (matura) in mathematics in 2010 by the students of the analyzed high school.

### 6. Ending

The applied synthetic measure allows to evaluate and organize analyzed institutions - in this case secondary schools in Wroclaw in terms of the three highlighted aspects. To measure the other two aspects of the analyzed schools, i.e. the process and personal one, there is necessity to access data which can be obtained from electronic registers of relevant classes. Furthermore, assuming that the assessment has characteristics of monitoring, which is to be carried out regularly, one can set the historical trajectory of the assessment and thus analyze the temporal changes in performance of teachers and schools (institutions).

### References


