Mathematical Connections Made by Teacher in Linear Program: An Ethnographical Study

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Abstract

Mathematical connection ability helps students to understand the concepts and the applications of mathematics, in this context, the teacher as an implementer of education has an important role to make a mathematical connection in their instruction. An ethnographic study was conducted to determine the teacher’s ability to make mathematical connections. A certified teacher with 30 years of teaching experience is observed and is interviewed to obtain the data. Data were analysed using thematic analysis. The findings show that the relationship between mathematics and everyday life arises as a mathematical connection in the form of different representations. When the teacher shows that a sentence can be another representation of a mathematical symbol, then that activity is a configuration of mathematical connection representation. In this study, the part-whole relationship is obtained not as a generalization but as a specific example. The relationship between ideas, facts, and concepts in mathematics appears in every construction, however, the process of knowledge construction is only carried out in the form of procedure and implication.

Keywords: mathematical connections, linear programs, different representations, ethnographic study

1. Introduction

Mathematics is a structured science in which concepts, principles, procedures are related to concepts, principles, and other procedures. The ability of students to make connections in mathematics itself is essential for understanding concepts (Anthony & Walshaw, 2009) as well as for applications outside the discipline of mathematics. The idea of mathematical connections with other subjects and real life began to be seen as a central point for mathematics education when the National Council of Mathematics Teachers included it in the curriculum (NCTM, 2000). Today, topics related to mathematics have attracted the attention of many researchers. These studies, among others, discuss the connections that arise in completing certain tasks (Jaijan & Loipha, 2012; Eli, Mohr - Schroeder, & Lee, 2011; Eli, Mohr - Schroeder, & Lee, 2013; Mhlolo, Venkat, & Schäfer, 2012; Islami, Sunardi, & Slamin, 2018; Santia & Kusumaningrum, 2017), conceptualisation of mathematical connections made by teachers (Businskas, 2008), connections between representations (Moon, Brenner, Jacob, & Okamoto, 2013; García-Garcia & Dolores-Flores, 2017), mathematical connections in solving-problems (Lockwood, 2011), and efforts to improve students’ mathematical connection skills (Yaniawati, Kariadinata, Kartasasmita, & Sari, 2017; Rohmatullah, 2018).
Based on the review, there are three facts: first, there has been no research on mathematical connections on the topic of linear programs; second, research on the mathematical connections that teachers make in their instruction has been examined by Businskas (2008) and Mhlolo, Venkat, & Schäfer (2012), however they only examined the relevance in mathematics and not discussed the relevance of mathematics to the real world. The link between mathematics and real-world problems is important to discuss because the relationship between the two can to increase interest and motivation in mathematics, contribute to the preparation of students to face the real world, develop positive attitudes towards mathematics, and develop understanding concepts (Lee, 2012; Karakoç & Alacaci, 2015; Bingölba & Coskun, 2016; Papadakis, Kalogiannakis, & Zaranis, 2017). They also did not identify the reasons for the teacher's actions. Identification is important to do in order to find out the obstacles and motivations of a teacher in making mathematical connections. The third fact is that there are no related studies, even though the fact that all studies focus on students' mathematical connection skills.

The importance of focusing on linear programs lies in two facts. First, the application of linear programs in daily life is comprehensive, such as allocating resources, planning production, scheduling workers, planning portfolio investments, and formulating marketing and military strategies (Matoušek & Gärtner, 2007). Second, mastery of linear programs requires students' ability to associate their initial knowledge such as systems of linear equations and convexity (Eiselt & Sandblom, 2007). Moreover, identifying and applying mathematics to contexts outside mathematics (the relationship between mathematics, other disciplines or the real world) and the interrelationships between ideas in mathematics are the basis of mathematical connections (Blum, Galbraith, Henn, & Niss, 2007).

All of these conditions have led the researchers to fill this gap with research that explores how teachers teach students mathematical connections to the topic of linear programming and what lies as behind the actions that teachers do.

### 2. Theoretical Framework

Businskas (2008) proposes a framework for identifying mathematical connections in teaching and learning practices. The model has five categories, namely different representations, part-whole relationship, implication, procedure, and instruction-oriented connections. Businskas found that the teacher had knowledge of mathematical connections as he defined them in his research, but the knowledge was implied mainly. Only a small proportion of teachers explicitly emphasise mathematical connections to students.

Important findings in a broader study conducted by Mhlolo (2011) on 20 observed subjects show that different representations, procedural connections, and instruction-oriented connections are the categories of mathematical connections that are often carried out from five categories of mathematical connections delivered by Businskas (2008), meanwhile, both part-whole relationships and implications only appear as much as 8% of all learning activities.

Businskas treats mathematical connections as the correct relationship between two mathematical ideas, A and B. Then, he can summarise the types of mathematical connections articulated by the teacher. Mhlolo (2011) then adds a table with the code on the model made by Businskas as follows.

**Table 1.** Forms of mathematical connections

<table>
<thead>
<tr>
<th>Form of Connection</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Different Representation</td>
<td>DR</td>
<td>Alternate representation: i.e. A is an alternate representation of B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equivalent Representation: i.e. A is equivalent to B - equivalent here is used to distinguish from representations in different forms and so refers to concepts that are represented in different ways within the same form of representation</td>
</tr>
<tr>
<td>2 Part-whole Relationship</td>
<td>PWR</td>
<td>A is included in (is a component of) B; B includes (contains) A, i.e. this is a hierarchical relationship between two concepts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A is a generalisation of B; B is a specific instance (example) of A. This is another kind of a hierarchical relationship</td>
</tr>
<tr>
<td>Form of Connection</td>
<td>Code</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------</td>
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<td>-------------</td>
</tr>
<tr>
<td>3 Implication</td>
<td>IM</td>
<td>A implies B (and other logical relationships), i.e. this connection indicates a dependence of one concept on another in some logical way. If... then...</td>
</tr>
<tr>
<td>4 Procedure</td>
<td>PR</td>
<td>A is a procedure used when working with object B</td>
</tr>
<tr>
<td>5 Instruction-oriented Connection</td>
<td>IOC</td>
<td>A and B are both prior knowledge concepts/skills that must be known to understand/learn C. This form of connection also includes an extension of what students already know – linking the new concept to prior knowledge</td>
</tr>
</tbody>
</table>

Rowland, Martyn, Barber, and Heal (2000) give an important finding of teachers' knowledge in learning practices. They produce findings that called as The Knowledge Quartet (KQ) which has four dimensions. The first dimension is the foundation, consisting of knowledge, beliefs, and understanding related to mathematics instruction. The second dimension, transformation, involves knowledge in action as shown both in planning for teaching and in the act of teaching itself. The third dimension, connection, involves how the teacher can see the relationships within and between lessons. It includes the sequence of learning material; the final dimension, accident and implementation of activities in the classroom that are not considered in teacher planning. Since this study wants to examine the practices of learning carried out in the classroom without regard to how the teacher prepares their instruction, then this study will discuss the first, second and third aspects (only in the act of teaching itself).

3. Method

This research is an ethnographic study because it aims to get a comprehensive picture of the natural state of a class, learn the role performed by the teacher in the classroom and behaviour related to that role. Many subjects studied by ethnographers are usually small, often only a few individuals, or one class (Fraenkel, Wallen, & Hyun, 2012). Qualitative research was conducted through the use of semi-structured interviews and thematic analysis was used to analyse the data.

The subject of this research is a certified teacher (named BE) working at one of the secondary school in Surakarta, Indonesia. She had 30 years of teaching experience where 12 years were spent teaching at the academic acceleration program. She also had obtained a bachelor’s degrees in mathematics education. This research concentrates on a single individual since the researchers attempt to observe everything within the setting or situation they are observing, in a sense they do not sample at all. What the researchers point out is a complete understanding of a particular situation. Conducting teacher interviews and observations enabled the researchers to develop an understanding of the interviewees' teacher knowledge and to begin viewing mathematical connection from the teacher's perspectives.

Data in this study were collected from interviews and observations. The researchers recorded the teacher's activities in the classroom during the linear program. In addition, the researchers also made field notes for observing teaching and learning activities in the classroom. Observations were made in two different classes and took place from the start of the linear program topic to completion. Data collected from the actions given by the teacher cannot be obtained only from the results of the observation. Therefore, the researchers also collected data through interviews conducted repeatedly at different times. The interview and observation instruments are used as procedures to check validity and reliability in qualitative research (Fraenkel, Wallen, & Hyun, 2012).

The data analysis method used in this study is a thematic analysis suggested by Braun and Clarke (2006; 2012). The analysis aims to identify patterns of meaning (themes) through a series of data provided by the answers to the research questions raised. According to Braun and Clarke (2006), themes capture something essential about data related to research questions, and represent several levels of response or meaning patterned in data sets. Patterns are identified through a rigorous process of data recognition, data coding, and theme development and revision. These methods are arranged in six phases, as follows, familiarise the data, generate the initial code, search for themes, review the theme, determine the name of the theme, and prepare reports. The analysis was based on a mathematical connection made by the teacher. The results of the
analysis are presented in a narrative method.

Quality criteria used in quantitative research, e.g. internal validity, generalizability, reliability, and objectivity, are not suitable to judge the quality of qualitative research (Korstjens & Moser, 2017). Qualitative researchers speak of trustworthiness. The criteria of trustworthiness are credibility, transferability, dependability, and confirmability (Denzin & Lincoln, 2017). Credibility—lasting presence during observation of deep interviews, gathering data at different times and using multiple methods of data collection i.e. interview and observation. Transferability—describing the learning activities regarding linear program and interaction between teacher and students in detail each meeting. Dependability—using external auditors to get an objective assessment, the accuracy of research data, level of data analysis, and matters relating to the relationship of problem formulation. Confirmability—making the research process as transparent as possible by clearly describing how data is collected, the procedures undertaken during the study, analysed, and the theory. Meanwhile, the issue of reliability addressed through replication because the researchers observed the teacher in the different class but the same grade.

4. Findings

In the context of findings, the teacher’s instruction is presented by identifying mathematical connections and supported by excerpt taken from interviews and observations. Furthermore, in the interview transcript, the researchers will be referred to as R, the subject BE will be referred to as BE, and the students referred to as S.

Based on the Indonesian Minister of Education Regulation Number 22 of 2016 (Kemendikbud, 2016), in the preliminary activities, the teacher must motivate students to learn in the form of benefits and application of the material in daily life and ask questions that relate previous knowledge to the material to be learned. Both of these activities are the two main types of mathematical connections delivered by Blum, Galbraith, Henn, & Niss (2007). In practice, the teacher starts a linear program by linking previous knowledge, namely linear inequality, the teacher does not appear to give students motivation to learn in the form of benefits and application of linear programs in daily life. The teacher connects the new concept to a linear program with the initial knowledge that students have that is drawing the feasible region from a linear inequality of two variables, this indicates that the teacher connects in the form of IOC. Unfortunately, this connection is still simple because it only connects knowledge about image shading, even though the prior knowledge of linear programs are more than that. Interesting facts are shown in the results of interviews with teacher.

Excerpt 1
R: How do you usually start a linear program?
BE: Submitting the title, then giving the main material to be given, as a whole the material is like, such a concept map, then determining the feasible region of an inequality system, and then, giving the overall subject matter.

Excerpt 2
R: In your opinion, what the initial knowledge should students have to study the linear program?
BE: How to draw lines, determine line equations, and eliminate systems of two-variable linear equations to find the intersection of two lines?

What the teacher delivered at Excerpt 1 is an activity that generally exists in the Minister of Education Regulation Number 22 the Year 2016. The teacher finally delivers in detail the initial knowledge needed by the student, after the researchers ask her directly (see Excerpt 2). It appears that the teacher actually has knowledge of mathematical connections, but this knowledge is not conveyed directly in their instruction. These results support the findings of Businskas (2008) which states that teachers know mathematical connections as defined in their studies, but that knowledge is largely tacit. The teacher only conveyed a little prior knowledge needed by students since she believes that the student will automatically know the relevance of the linear program in real life when working with the problem, which is what makes the teacher decided not to convey it before the teaching process begins.
On the other hand, mathematical connections in the form of DR occur when the teacher explains that a function can be expressed in a graphical (as shown on P.1.2 in Fig. 1). However, the teacher does not explain that the two are related to each other. Furthermore, when the teacher explains the steps to draw a line, the teacher explains how to determine the coordinates of the point by making a table of values (see P.1.1 in Fig. 1). However, the teacher does not emphasize that the value table represents the point coordinates of a graph. In P.1.3 Fig. 1, it appears that the teacher does not draw the left arrow (on the X-axis) and the down arrow (on the Y-axis). The line in the Cartesian diagram must be drawn as a line that has two arrow directions; this is because the line represents a real number which has an interval $(-\infty, \infty)$. When researchers conduct further interviews with teacher, researchers find the fact that the teacher knows about this fact. However, it appears that the teacher considers the error is not a big problem because it is only a writing error.

Mathematical connections in the form of PWR occur when the teacher explains about the steps to draw the inquired line $3x + 2y = k$, this is seen in the following excerpt:

$k$ is any number that fulfills... so, the probe is taken from the target function, then the inquired line is drawn, to draw the line of inquiry, we have to know what the value is $k$, to determine the value of $k$, we take any point here (inside the feasible region), insert it so that it gets $k$... we can also take $k$ multiples of 3 and 2, for example, 6... Why? Because the lines are in shape $3x + 2y$ equals with... $3x + 2y$ equals with what... those are parallel lines...

Even though the teacher has tried to convey that line equation $3x + 2y = k$, representing parallel lines, but the teacher lacks emphasis that $3x + 2y = 6$ is a specific form of equation $3x + 2y = k$. It appears that the teacher immediately provides information about the meaning $3x + 2y = k$, without giving opportunity or directing students to relate their knowledge about the concept of parallel lines.

A mathematical connection in the form of PR occurs when the teacher explains the procedure of drawing a line (see Excerpt 3), determines the coordinates of the cut point (see Excerpt 4), and completes the linear program story problem (see Excerpt 5).

Excerpt 3

BE: Do you still remember, in grade X, to draw a line we need at least what point?
S: Two...

Excerpt 4

BE: What point B intersects what line with what?
S: $2x+y=8$ and $x+3y=9$

BE: Determine the $x$, and the $y$ then eliminate it...
S: Ooohh...
Excerpt 5
BE: So, to solve the story problem, what should we do?
S: Drawing the completion area ... making pictures ...
BE: Yes ... draw the feasible region, determine the corner point, the final destination determines the problem-solving.

Of the three connections presented, only in Excerpts 3 and 5, the teacher lets students associate the knowledge they have whereas, in Excerpt 4, the teacher immediately tells what initial knowledge students need. Our study also found that some of the PR submitted by the teacher contained mathematical connections in the form of DR, as shown in the following excerpt
Excerpt 6
BE: (the teacher provides a table of values that have not been filled) Okay, if x worth zero then y?
S: Two...
BE: If y worth zero then y?
S: Three...
BE: So, we have got (0,2) and (3,0)
Excerpt 7
BE: To find the maximum and minimum values; enter the corner points into the objective function (then the teacher provides a table of function values)
S: (forward to the class, fill in the value table provided by the teacher)
In Excerpt 6, it appears that the teacher is explaining the steps to draw a line while in Excerpt 7 the teacher explains the steps to find the optimum value. Not only in the PR but also the IM, the teacher gives students the opportunity to construct their knowledge by directing students to find the relationship between the boundaries of the functional constraints and the shape of the feasible region, this can also be seen in the following Excerpt 8
Excerpt 8
BE: Now pay attention, from this graph, this, and this, what conclusions do you get? If the problem functions are less than the same, how about the feasible region?
S: Closed
BE: If the constraint function has a bigger sign equal to?
S: Open
The teacher gives a number of graphs that display several different shapes of the feasible region then asks students to observe the patterns in the graphs. After that, the students were asked to draw a conclusion based on their observations.

5. Discussion

The following will discuss the forms of mathematical connections made by the teacher on linear program material. The forms of mathematical connections that will be discussed consist of IOC, DR, PWR, PR, and IM

The interview results showed that the teacher knows mathematical connections in the form of IOC, but this knowledge was not conveyed directly in their instruction. These results support the findings of Businskas (2008) which states that teachers know mathematical connections as defined in their studies, but that knowledge is mostly not disclosed.

The implied submission of the IOC is because of the teacher's belief that the student, by itself, will know the prior knowledge after completing the module book. These results support the findings of Rowland, Martyn, Barber, and Heal (2000) about The Quartet Knowledge. In the first dimension, it said that the teachers were used their knowledge, belief, and understanding in their instruction.

It was also seen that the teacher's beliefs influenced the teacher decisions in their instruction. As stated by Harste & Burke (1977) and Kuzborska (2011), the teacher's decisions about their instruction based on their beliefs about teaching and learning. Furthermore, the teacher, in his interview, also added that providing the relevance of a linear program in real life before the teaching process begins would take a lot of time. As stated by Henson (2001), teacher beliefs influence classroom management. The time management carried out by the teacher is based on the belief
that the relevance of a linear program no need to be delivered at the beginning of learning, however, this is not the right thing because of the importance of encouraging students' readiness in learning activities (Puteri, 2018). In addition, students will have strong motivation and encouragement to absorb learning, so that it can attract students' attention (Ahn, 2014; Gubi, Platton, & Nelson, 2008; Linnenbrink & Pintrich, 2002).

The teaching experience in the 12-year acceleration program made the teacher want to complete the material; the teacher was impatient if she waited for students to find their concepts and preferred to notify students directly, it indicates that teacher see students as passive recipients (OECD, 2009). BE have a total of 30 years of teaching experience which can be said to be experienced teacher (Gonzalez & Carter, 1996; Berger, Girardet, Vaudroz, & Crahay, 2018). The belief about the transfer of knowledge possessed by teachers makes the findings of Berger, Girardet, Vaudroz, & Crahay which states that the more experience the teachers have, the belief in constructivism will be better than knowledge, does not happen to BE.

The researchers assume that the 12 years experience in the academic acceleration program makes the teacher have the belief about student-centered instruction and about the time management. This result also supported by the findings of Ünal & Ünal (2012) which contradict with the findings of Berger, Girardet, Vaudroz, & Crahay. Ünal & Ünal stated that years of experience play a significant role in the teacher's beliefs and attitudes in strengthening their classroom management, in addition, teachers with years of teaching experience are found to support maximum teacher control (interventionism).

This form of connection appears implicitly when the teacher explains that a function can be expressed in graphical form. Another type of DR can be seen when the teacher describes the relationship between a value table and a graph; we might interpret this form of connection as PR because the compilation of value tables is the first step in producing a graph. Even so, Businskas (2010) explains that when talking about graphics, teachers talk about the meaning of various aspects, on the contrary, much of what they say about algebra focuses on simplification and solving.

In the form of DR, the researchers also found that there was an act of letting "small" mistakes made by students, namely when describing arrows on the Cartesian diagram, this was because the teacher thought that the error was not a severe problem. However, this type of error may not jeopardise the current learning objectives, but it may hamper future learning, this is not the teacher's full consideration. If the teacher ignores this type of error, it can take root over time, resulting in big misunderstandings (Li, 2006). Given the need for students to know the connection in mathematics (NCTM, 2000), students need to understand that the two-way arrow symbol in the Cartesian plane is another representation of real numbers.

In the mathematical connection of the form of the PWR, the teacher returns it directly, without giving students the opportunity to relate their knowledge of the concept of parallel lines. The teacher should not tell explicitly and exposing similarities among existing cases; this action can cause the participants not to be too familiar with the knowledge they expect they can make (Mhlolo, 2011).

The PWR obtained in this study is a specific example. Meanwhile, PWR findings from Mhlolo (2011) are in the form of generalisations. However, both the results of Mhlolo's findings and these findings indicate that this mathematical connection in the form of PWR is less common than other forms of mathematical connections. It can be understood, remembering examples and generalisations are usually made after making a series of observations. The teacher can explore this connection more by explaining that corner points are only part of a set of points in the feasible region, Businskas (2008) refers to this as inclusions: an object is a component of a larger idea or set.

If at IOC, DR, and PWR, mathematical connections are conveyed directly (students are not given the opportunity to construct knowledge), in the PR, the teacher has been seen to invite students to construct their knowledge. That is the right action because if they construct their knowledge, they will better understand new knowledge (Ültanır, 2012; Bada, 2015). Moreover, the activity of constructing knowledge proves to be more effective in improving student learning achievement than receiving knowledge (Kim, 2005).
The findings also indicate that several PR presented by the teacher contain mathematical connections in the form of DR. When explaining the steps to draw lines and steps to find the optimum value, the teacher indirectly conveys that a function can be expressed in the form of a value table (see P.1.1 in Fig. 1). Thus, this study has added the findings of Businskas (2008), because if previously DR was separated from PR, then in this study, it was found that in PR there was a DR.

The results of this study again support the findings of Mhlolo (2011) that in addition to PWR, the form of connection in the form of IM also rarely appears compared to the form of mathematical connections in the form of PR, DR, and IOC. Both IM and PWR, only appear once in linear programs instruction.

Furthermore, in this form of connection, the teacher looks to direct students to find that if the constraint function limit has less than equal to \( x \geq 0 \) and \( y \geq 0 \), as a result, the feasible region that is formed is a closed area, on the contrary, if the constraint function has more than the same sign, the result is that the feasible region formed is an open area. The teacher's decision to not immediately inform the mathematical connection that exists but to invite students to find it for themselves is the right action, because each student will try to understand all the information they get, and they will understand the information, therefore students need to actively build knowledge in their own minds (Piaget, 1973; Ültanır, 2012; Bada, 2015). Moreover, teachers need to support students autonomy and provide a strong structure to facilitate students constructing the relevance of the learning carried out (Berger, Girardet, Vaudroz, & Crahay, 2018).

In addition to the five categories presented by Businskas, this study also found that the teacher made mathematical connections when helping students determine the inequality in the story problem. The teacher emphasizes that the phrase "minimal" indicates a sign that is greater than the same, while the sentence "maximise" indicates a smaller sign equal to.

Businskas (2008) states that different representations occur when the same concept can be expressed in two or more symbolic ways (algebraic), graphics (geometric), pictorial (diagram), manipulative (physical object), verbal description (spoken), written description. This statement about the written description makes researchers assume that the activity carried out by the teacher when showing how a sentence is another representation of a mathematical symbol, is a mathematical connection activity in the form of DR. These results appear as assumptions because, in the Businskas study, there is no further information about the written description, this is understandable because Businskas' research is free in the area of relevance in mathematics. Meanwhile, research that also bases research on Businskas' research results, namely Mhlolo, Venkat, & Schäfer (2012), also focuses on the relevance in mathematics and on the DR, making researchers have difficulty understanding the meaning of written description in Businskas's research.

6. Conclusion

This study analyses the teacher's actions in teach mathematical connections and the teacher's reasons for each action given. The results of this study have supported the findings of Businskas (2008), which indicates that the teacher has knowledge of mathematical connections, but that knowledge is mostly not conveyed. The findings indicate that this arises because of the teacher's assumption on that the emphasis on connections will be time-consuming.

Furthermore, the results of the study also show that there is a mathematical connection in an inappropriately, when showing that straight-line images are other representations of line equations of two variables, and when describing lines as other representations of a set of real numbers. This inappropriate mathematical connection is suspected because "small things" such as the meaning of representation of a mathematical symbol in an image are often ignored and do not see mathematics as a connection. Teacher education institutions should emphasise these "small things", to be delivered by prospective teachers, given the importance of connection in mathematics for students to know.

Of all the categories, the PR and IM is a connection that is conveyed through questions (students are given the opportunity to construct their knowledge). The results of this study also
support Businskas (2008) statement that some DRs look like PR, besides that, this study also found that in PR there is a DR.

As the results of Mhlolo (2011) study, in this study, the form of mathematical connections in the form of Implication and PWR are also things that are rarely raised by teacher, each of which occurs only once during teacher instruction. Furthermore, these findings have broadened the findings of Businskas (2008) and Mhlolo, Venkat, & Schäfer (2012) who both discussed the delivery of mathematical connections made by teachers. The results of this study also indicate the teacher also conveys a connection when emphasising that the word "minimal" indicates a greater than or equal to sign, while the sentences "maximal" indicates a less than or equal to sign. This connection did not appear in Businskas and Mhlolo's research because their research focused on connections in mathematics.

References


